The illustrative examples of the elementary short questions recommended by the Ad Hoc Committee on the Extended Part of Senior Secondary Mathematics (Enhanced Measures for Catering Learner Diversity)

Elementary short questions in Module One

1. Let $y = e^{\frac{x^2}{2}}$.
   (a) Using trapezoidal rule with 5 sub-intervals, estimate $\int_0^2 y \, dx$.
   (b) Find $\frac{d^2y}{dx^2}$.
   (c) Determine whether the approximate value obtained in (a) is an over-estimate or an under-estimate. Explain your answer.

2. Let $\frac{dy}{dx} = \frac{(1+\ln x)^3}{x} + \frac{2x}{x^2+1}$. It is given that $y = 0.25$ when $x = 1$. Using integration by substitution, express y in terms of x.

3. (a) Expand $2 - e^{2x}$ in ascending powers of x as far as the term in x^2.
   (b) The coefficient of x^2 in the expansion of $(kx + 2)^2(2 - e^{2x})$ is -23, where k is a constant. Find k.

4. Let C be the curve $y = 12x - 2x^2$. Find
   (a) $\frac{dy}{dx}$;
   (b) the equation of the tangent L to C, where L is parallel to the straight line $8x + y = 0$.

5. It is given that $y = e^{kx}(3x + 1)$, where k is a constant.
   (a) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of k.
   (b) If $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, find k.

6. Let C be the curve $y = x^3 - 5x^2 + 8x - 4$. Find
   (a) the extreme point(s) of C,
   (b) the area of the region bounded by C and the x-axis.

7. When a spherical balloon is being inflated, its radius r cm after t s is given by $r = 2t^{\frac{1}{2}}$, where $0 \le t \le 9$. The volume and the surface area of the balloon are V cm³ and A cm² respectively. Find
   (a) the value of $\frac{dV}{dt}$ when $t = 4$,
   (b) the value of $\frac{dV}{dA}$ when $t = 4$.

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8. It is given that the relation between x and y can be modelled by $y = Ae^{kx}$ where A and k are constants, and $A > 0$. The following table shows some values of x and their corresponding values of y.

x	2	4	6	8
y	120 000	12 000	1 200	120

(a) Express $\ln y$ as a linear function of x.
(b) Estimate the values of A and k graphically, correct to 1 decimal place.
(c) Using the results of (b), estimate the value of y when $x = 10$.

9. Let A and B be two events. B' is the complementary event of B. Suppose that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B') = 0.7$.
(a) Find $P(A \cap B')$.
(b) Are A and B independent? Explain your answer.
(c) Find $P(B|A)$.

10. Susan has 6 paperback books and 2 hardback books. She randomly chooses 4 of these books for donation. Let X be the random variable representing the number of hardback books she chooses. Find
(a) the probability distribution of X,
(b) $E(X)$ and $E(X^2)$,
(c) $\text{Var}(X)$ and $\text{Var}(3X + 2)$.

11. Assume that a random variable $X \sim B(6, p)$. It is given that $E(X) = 0.9$. Find
(a) $\text{Var}(X)$,
(b) $P(X = 4)$,
(c) $P(X < 3)$.

12. The weights of lemons follow a normal distribution with a mean of μ g and a standard deviation of σ g. It is given that 1.97% of the lemons are heavier than 115 g and 3.92% of the lemons are lighter than 85 g. Find
(a) μ and σ,
(b) the probability that the weight of a randomly chosen lemon lies between 90 g and 110 g.

13. The number of telephone calls received by a shop per minute follows a Poisson distribution with a mean of 0.3. Find
(a) the probability that there are no calls in a minute,
(b) the probability that there are fewer than 3 calls in a minute,
(c) the probability that there are more than 2 calls during a two-minute period.

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14. The lifetime of a certain type of battery is assumed to follow a normal distribution with mean μ hours and standard deviation σ hours. A survey is conducted to estimate μ. From a random sample of 49 batteries drawn, a 95% confidence interval for μ is found to be $(18.6,\ 27.8)$.

   (a) Find σ.
   (b) Find the sample mean.
   (c) If another random sample of batteries is taken, find the least sample size such that the width of the 90% confidence interval for μ is less than 8.

15. Electric folding fans are produced by two production lines X and Y. It is given that 5% of all electric folding fans malfunction and 4% of electric folding fans produced by production line X malfunction. Among the electric folding fans which can function properly, 60% of them are produced by production line Y. Suppose an electric folding fan is randomly selected.

   (a) Find the probability that the electric folding fan is produced by production line Y and can function properly.
   (b) Find the probability that the electric folding fan is produced by production line X.
   (c) Given that the electric folding fan is produced by production line Y, find the probability that it can function properly.